Supplementary Material: A MEMS-based Foveating LIDAR to enable Real-time Adaptive Depth Sensing

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1 Derivations

Design	Volume	FOV	SNR
Retroreflection	$\frac{\pi\ u\ w_o^2}{12}$	$=$ MEMS FOV ω_{mirror}	$\frac{atan(\frac{w_o}{2Z})}{\omega_{\text{laser}} Z tan(\frac{\omega_{\text{laser}}}{2})}$
Receiver array	$u\ A^2$	$\min(2\ atan(\frac{A}{2u}), \omega_{mirror})$	$\frac{1}{2\ Z\ tan(\frac{\omega_{laser}}{2})}$
Single detector . *Conventional* $(u \geq f)$ ***Ours*** $(u < f)$	$\frac{\pi\ u\ A^2}{12}$	$\min(2atan(\frac{A(Z-f)\|\|\frac{Zu-fu-fZ}{Z-f}\|\|}{2ufZ}), \omega_{mirror})$	$\frac{1}{4Zatan(\frac{A(Z-f)\|\|\frac{Zu-fu-fZ}{Z-f}\|\|}{2ufZ})tan(\frac{\omega_{\text{laser}}}{2})}$

Table 1. Receiver models.

Here we derive all the formulae in Table 1 for the three designs. We have provided the ray diagrams of the designs in Fig. 1 and we have reproduced Table 1 here.

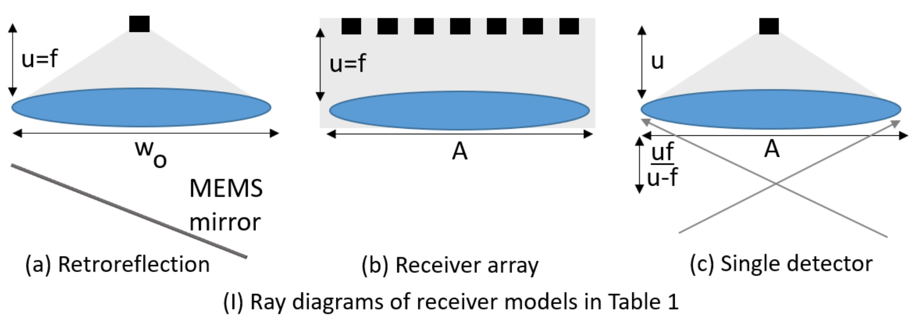

Fig. 1. Ray diagrams of designs.

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* Equal Contribution

1.1 Volume

For the retroreflection and single detector, the volume of the camera is a cone whose vertex is the location of the single detector. From the ray diagrams and from the equation of the volume of a cone, this is easily seen to be $\frac{\pi u w_o^2}{12}$ for the retroreflector and $\frac{\pi u A^2}{12}$ for the single detector. For the receiver array the volume is the entire enclosure, given by the volume of a cuboid, $u * A * A$.

1.2 FOV

The retroreceiver has exactly the FOV of the mirror, by definition. From 1(b), the FOV of the receiver array is given by the vertex angle of the cone at the central pixel, given by $2atan(\frac{A}{2u})$, bounded by the FOV of the mirror. This assumes the receiver and transmitter are close enough to ignore angular overlap issues.

To find the FOV of the single detector, consider the diagram in Fig. 3, where the single detector is focused on the laser dot at distance Z from the sensor. From similar triangles, the kernel size is given by first finding the in-focus plane at $u^{'}$ from the lens equation

$$\frac{1}{f} = \frac{1}{u^{'}} + \frac{1}{Z} \tag{1}$$

and so $u^{'} = \frac{fZ}{(Z-f)}$. From the two vertex shared similar triangles on the left of the lens, we now have an expression for the kernel size

$$kersize = abs(u - u^{'}) * \frac{A}{u^{'}} \tag{2}$$

Substituting the value of $u^{'}$, we get an expression for $kersize =$

$$abs(u - \frac{fZ}{(Z-f)}) * \frac{A}{\frac{fZ}{(Z-f)}} = \frac{A(Z-f)\|(Zu - fu - fZ)\|}{fZ\|Z-f\|} = \frac{A(Z-f)\|\frac{Zu-fu-fZ}{Z-f}\|}{fZ} \tag{3}$$

From the figure, the FOV, given by kernelangle is

$$2atan(\frac{kersize}{2u}) = 2atan(\frac{A(Z-f)\|\frac{Zu-fu-fZ}{Z-f}\|}{2ufZ}) \tag{4}$$

1.3 SNR

From Fig. 2, the power from the laser decreases with distance. This is just fall-off from the source, and we represent it here as the area of the laser dot on a fronto-parallel plane. From the figure, this can be calculated from simple trignometry as $2Ztan(\frac{\omega_{laser}}{2})$, and we use the reciprocal for the SNR as $\frac{1}{2Ztan(\frac{\omega_{laser}}{2})}$.

1. Receiver array: The receiver array is assumed to capture all the available radiance from the laser dot, and so the SNR is exactly the same as the power

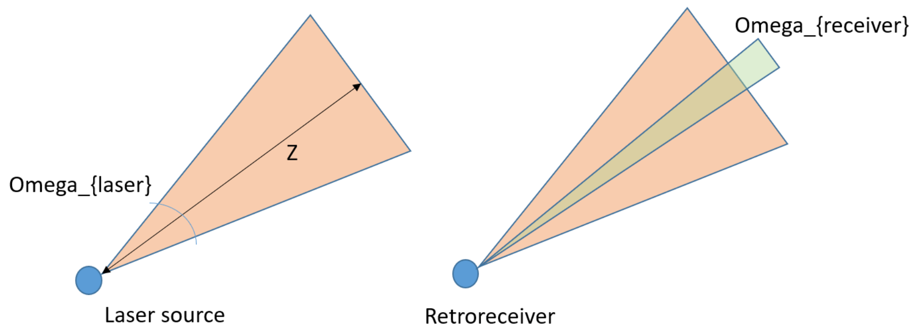

Fig. 2. Retroreflective SNR.

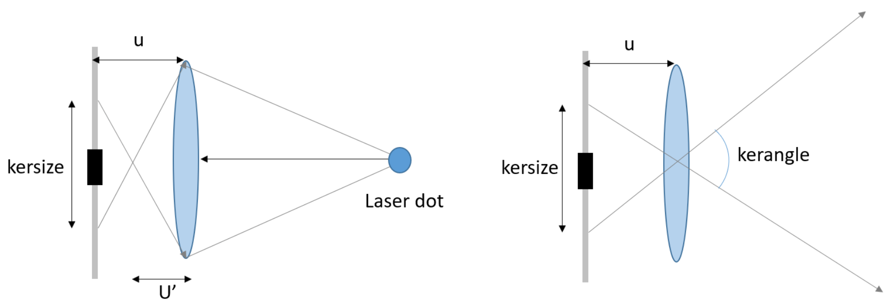

Fig. 3. Single detector SNR.

fall-off described above as $\frac{1}{2Ztan(\frac{\omega_{laser}}{2})}$.

2. Retroreflection: As can be seen in the right of Fig. 2, the ratio of the received angle to the transmitted angle gives the fraction of the received radiance from the laser dot. From Fig. 1(a), the single detector receives parallel light of width w_o. For any particular depth Z therefore, the angle subtended by this width at the sensor decreases and is given by $\omega_{receiver} = 2atan(\frac{w_o}{2Z})$ and the fraction of the fall-off received is given by $\frac{2atan(\frac{w_o}{2Z})}{\omega_{laser}}$. Multiplying this with the fall-off above gives $\frac{2atan(\frac{w_o}{2Z})}{\omega_{laser}} * \frac{1}{2Ztan(\frac{\omega_{laser}}{2})} = \frac{atan(\frac{w_o}{2Z})}{w_{laser}Ztan\frac{\omega_{laser}}{2}}$. Note this assumes that $\omega_{receiver} < \omega_{laser}$, and if this is not the case then a max function must be added so that $\omega_{receiver}$ does not exceed ω_{laser}.

3. Single detector: We just reduce the fall-off by the kerangle calculated before, and therefore the SNR is $\frac{1}{kerangle*2Ztan(\frac{\omega_{laser}}{2})} = \frac{1}{4Zatan(\frac{A(Z-f)\|\frac{Zu-fu-fZ}{Z-f}\|}{2ufZ})tan(\frac{\omega_{laser}}{2})}$